

# Cryptography

## Problem set 8 - 12-13 VI 2013

**Problem 1** Consider an additive group  $Z_n$ . An element  $y \in Z_n$  is a quadratic residue if and only if there exists an  $x \in Z_n$  such that  $x^2 = y \pmod n$ .

- What are the quadratic residues in  $Z_p$  for  $p$  – an odd prime?
- Let  $n = pq$  be a product of two odd primes  $p$  and  $q$ . What are the quadratic residues in  $Z_n$ ?
- Let  $n$  be an even integer. What are the quadratic residues in  $Z_n$ ?

**Problem 2** Let  $N = pq$  with  $p, q$  distinct, odd primes. Prove that if  $x \in QR_N$  (quadratic residues over  $Z_N^*$ ) then  $[x^{-1} \pmod N] \in QR_N$ , and if  $x \in QNR_N^{+1}$  (quadratic non-residues) then  $[x^{-1} \pmod N] \in QNR_N^{+1}$ .

**Problem 3** Let  $N$  be the product of 5 distinct odd primes. If  $y \in Z_N^*$  is a quadratic residue, how many solutions are there to the equation  $x^2 = y \pmod N$ ?

**Problem 4, 5, ...** to be continued