SCV end to end voting over the Internet

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Abstract. We present *Scratch, Click & Vote* remote voting scheme. The scheme is end-to-end verifiable and allows for voting over the Internet. It guarantees security against malicious hardware and software used by a voter; a voter's computer does not get any knowledge about the voter's choice. Moreover, it can blindly change the voter's ballot with a small probability only. As a side result, we present a modification of the ThreeBallot that eliminates Strauss'-like attacks on this scheme.

Keywords: Internet voting, e-voting, E2E, verifiable voting scheme, ThreeBallot, Punchscan

1 Introduction

There are two main scenarios of e-voting: advanced voting procedures at polling places and remote electronic voting.

Polling station voting Recently it has became evident that badly designed e-voting machines can be extremely dangerous to a voting process [8,10,14]. Fortunately, a number of end-to-end auditable voting systems (*E2E*) has been presented recently. Interestingly, some recent designs implement *electronic voting* without any electronic voting machines [3,6,5,4]. Moreover, for these schemes each voter gets a receipt, which may be used to check if the voter's ballot has been included in the tally. It is also possible to verify correctness of the results. On the other hand, the receipt cannot be used even by the voter to prove how she voted. So they cannot help to sell or buy votes.

Internet voting The Internet voting has not much in common with polling station scenario. At the polling station it is relatively easy to preserve voter's privacy; coercion and vote buying is hard to hide. Another source of problems is that a voter must use some electronic device. There is no convincing argument why a voter should blindly trust this device. Malware can endanger integrity of the elections as well as privacy of the voter.

In case of remote voting one has also to deal with remote voter identification. Fortunately, it can be solved in many ways, depending on a situation. For national elections one can use advanced electronic signatures, especially if supported by personal, government-issued ID cards, or a novel technique described in [2]. For other elections logins and passwords seem to serve well their purpose.

In this paper we are concerned with an *E2E* systems for remote voting over electronic networks. We assume that the electronic devices used by a voter might be infected by malicious code, and that voter's privacy and election integrity must be guaranteed in a verifiable way.

Three important ideas concerning *E2E* voting systems have been presented during the last few years: Prêt à Voter, Punchscan, ThreeBallot (and related schemes). All of them are dedicated to paper-based elections at polling stations. Recently, Punchscan and Prêt à Voter have been adjusted to mail-in voting [**?**]. Since these methods are closely related to our scheme, we recall them briefly.

Prêt à Voter [6]. A voter, say Alice, obtains a ballot which consists of two parts. The left part contains the official list of the candidates, altered by applying a circular shift by x positions, where x depends on the ballot. The right part contains boxes where Alice can put the ×-mark. In order to vote, she puts the ×-mark in the row that contains the name of her favorite candidate on the left side. On the right side there is a kind of ballot serial number S that is used for decoding Alice's vote (namely, for reconstructing the shift value x). The serial number is also included in the voting receipt obtained by Alice. After making her choice, Alice separates both parts. The left part goes to a shredder, while the right part is scanned and entered to the system.

$Candidate$	S
2 Jerry	
3 Edgar	
0 Ervin	
1 Donald	

(a) Prêt à Voter ballot with shift $x = 2$

$Candidate$	S
2 Jerry	
3 Edgar	
0 Ervin	×
1 Donald	

(b) A vote for candidate number 0 (Ervin)

S
×

(c) The receipt of the vote

Fig. 1. Ballot example for Prêt à Voter scheme

Punchscan [3]. The original ballot design of Punchscan is quite different from Prêt à Voter, however it has been shown in [20] that the crucial mechanisms of Punchscan can be used together with Prêt à Voter ballot layout.

The key issue is that Punchscan offers a complete back-end to perform *E2E* verifiable elections. Similar back-end is also used in Scantegrity [5] and Scantegrity II [4]. The values that are used in the ballot construction are committed and can be verified. The verification process is twofold and consists of a pre-election audit and a post-election audit. If the authority responsible for preparing ballots passes both audits, then with an overwhelming probability the integrity of the elections is guaranteed.

ThreeBallot [17]. This scheme, presented by R. Rivest, is particularly appealing despite of certain privacy weaknesses [7]. A voter, Alice, obtains a sheet of paper consisting of four parts. The leftmost column contains the list of candidates (no shift is used). The next three columns are used to mark her choice. If she wants to vote for a candidate V, then she puts two marks × in the row containing the name of V, while she puts exactly one × mark in all remaining rows. After Alice makes her choice, all three columns (ballots) are separated and cast into a ballot box. As a receipt Alice obtains a copy of one of the columns/ballots of her choice (but the system does not know which one).

$Candidate$	A	B	C
0 Ervin			
1 Donald			
2 Jerry			
3 Edgar			

(a) An empty ballot

$Candidate$	A	B	C
0 Ervin	×		×
1 Donald			×
2 Jerry	×		
3 Edgar		×	

(b) A vote for Ervin

A
×
×

(c) Exemplary receipt

Fig. 2. The ThreeBallot scheme

Internet voting schemes. So far, the schemes designed by the academic community do not fulfill all security demands. The most acute problem is that almost all schemes ignore the fact that electronic voting equipment should be considered as a potential adversary ([13,15]). Meanwhile, potentially the most dangerous in remote voting systems is the equipment on the voter's side. Voters' machines can be infected with malware that reveal the voter's preferences or even change the encrypted ballot cast by the voter.

1.1 Design goals

The main goal behind design of *SC&V* is to get an *E2E* scheme that would be acceptable for casting votes over Internet. As in the previous chapter we demand all key security requirements to be satisfied. In particular, the scheme must be secure without assuming that the voter's PC or any of its components is trustworthy. This setting corresponds to the real-life situation, where even PC equipped with strong anty-virus protection and a trust oriented operating system cannot be fully trusted.

The scheme presented in the previous chapter satisfies the mentioned goals. However it fails usability and transparency issues: it is so complex that explaining the security mechanism even to a well educated voter is a challenge.

We present the scheme which is:

human verifiable: a receipt obtained by a voter is human-readable and easy to examine by a moderately educated voter,

voter friendly: a voter (i.e. her computer) needs not to perform any complicated (and hard to understand by an average voter) operations like: re-encryption, getting a blind signature, executing oblivious transfer protocol etc.

malware immune: integrity of the elections and privacy of votes do not rely on any assumption on trustworthiness of the equipment used by the voter,

efficient: computational overhead as well as communication volume are low. (Note that the scheme from the previous chapter requires many ciphertexts for each single candidate. Moreover, at least two ballots has to be fetched. The overhead due to the number of candidates is negligible for the scheme presented in this chapter.)

1.2 A short overview of *SC&V*

In the next sections we provide a full, detailed description of *SC&V*(*Scratch, Click & Vote*). Here we present the idea standing behind the scheme. *SC&V* can be described as a layered design in which we combine a number of techniques/tricks:

Version 1: We start from a straightforward Internet-version of the ThreeBallot: a voter needs to fill in a voting card using an application run on her PC. The filled ballot is then sent to a voting server (*Proxy*).

Version 2: In order to balance the number of × marks in the columns (in order to make the scheme immune against Strauss'-like attacks [7,11,18,19]) we add another column and another ×-mark ("FourBallot" design).

Version 3: Since the voter's PC knows exactly the voter's choice – we introduce a coding card which is prepared by a *Proxy* (see the diagram below). The coding card hides from the PC the meaning of the voter's choices (every candidate is "clicked" exactly once). Moreover, the coding card is constructed in a way that the possibility of modification of voter's choice by her PC is reduced.

Version 4: Since *Proxy* still knows the voter's choice, we introduce another server – Election Authority (EA), which is responsible for preparation of ballots. In this way *Proxy* does not know the choice, while Election Authority does not know who cast a certain vote.

Election Authority prepares ballots in a similar way as for the Punchscan scheme. Together with the ThreeBallot mechanisms this ensures verifiability of the voting process.

Under the assumption that *Proxy* and *Election Authority* do not collude, *SC&V* offers verifiable Internet voting with unconditional integrity and full privacy of a voter.

1.3 Ideas overview

Ballots and coding cards The voting process is a combination of paper based protocols. In order to vote one has to get additional information that remains hidden from the computer used for vote casting. This information might be obtained by the voter during voter's in-person registration, mailed to her home address or sent over an independent electronic link. The method applied can be tailored according to the security demands of specific elections. In case of small scale elections or elections of limited importance one can use emails with CAPTCHA to sent information readable for a human voter, but hard to read by the voter's computer. Vote casting is done via electronic networks (no matter which registration method was used).

There is an *Election Authority* (*EA*) and a *Proxy*. *EA* prepares the *ballots* while *Proxy* prepares *coding cards*. There is an *Auditor* which is responsible for pre- and post-election audits.

In this section we describe the scheme from the point of view of a voter Alice. For the sake of simplicity of exposition we assume that there is a single race where the voter has to choose one out of m candidates (the pictures presented below depict the case of $m = 4$).

Ballot layout. In order to cast a vote, Alice needs a *ballot* and a *coding card*. The **ballot** is prepared by *EA*, it consists of the following values covered by a scratch surface:

- list of candidates permuted with some random permutation π. Later we shall represent $\pi = \pi' \circ \pi''$ where π', π'' are chosen at random.
- ballot serial number S_l,

Candidate	A	B	C	D
2 Jerry				
3 Edgar				
0 Ervin				
1 Donald				
S_l				

Fig. 3. A ballot with $\pi(i) = i + 2$

- four *confirmation tokens*: A, B, C, D – one per column. They are prepared in a special way that is described below.

The **coding card** is prepared by *Proxy* and consists of:

- four columns. In each row there is exactly one mark `Y` standing for YES, and 3 marks `n` standing for NO. The placement of `Y` in each row is random and independent from the other choices.
- S_r, which is the coding card serial number.

	n	Y	n	n
	n	Y	n	n
	Y	n	n	n
	n	n	n	Y
	S_r			

Fig. 4. A coding card

1.4 Voter's point of view

Alice obtains both: the ballot and the coding card (during in-person registration, by mail or by email, depending on election settings). Let us note that Alice gets exactly one ballot, but she is allowed to have as many coding cards as she likes. Moreover, we assume that there are many Proxies in the system[1], so Alice can easily find one she trusts and gets coding cards from this Proxy.

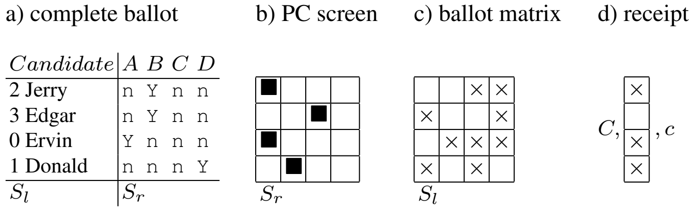
Alice lays the ballot and the coding card side by side and thus obtains a complete ballot. A complete ballot (which Alice may put on her desk) may look as follows:

Alice visits an election website operated by the *Proxy*. She authenticates herself with appropriate authentication method (login and password, electronic signature . . .). She clicks on the screen in the following way:

- she clicks on the position of `Y` in the row corresponding to the candidate that she votes for,
- in each of the remaining rows, she clicks on one of the positions of `n`'s.

The *Proxy* commits to Alice's clicks (the commitment is passed to *EA*), then Alice enters coding card serial number S_r. The *Proxy* checks S_r and then transforms the choice of the voter into an internal form called *ballot matrix*:

[1] Moreover, a "decoy service" can be introduced – then Alice may obtain many different but fake coding cards with the same serial number – in order to cheat a coercer or a vote-buyer.



$Candidate$	A	B	C	D
2 Jerry	n	Y	n	n
3 Edgar	n	Y	n	n
0 Ervin	Y	n	n	n
1 Donald	n	n	n	Y
S_l	S_r			

■			
		■	
■			
	■		

		×	×
×			×
	×	×	×
×		×	

×
×
×

Fig. 5. SCV voting scheme from voter's perspective:
a) A complete ballot consisting of a ballot aligned with a coding card.
b) A vote for the candidate *0 Ervin* as seen on the screen.
c) A ballot matrix - an internal ballot representation derived deterministically from the complete ballot and voters choice/clicks.
d) A receipt chosen by a voter - in this case it corresponds to the third column of ballot matrix.

Proxy puts × mark for each n which has not been used yet (this transformation depends deterministically on the positions of Y's and n's and the voter's choice). So, for a row with the candidate chosen by the voter *Proxy* puts three × marks, while in each row corresponding to different candidates, there are only two marks ×. Note that *Proxy* knows which row corresponds to the vote cast. On the other hand, due to the random permutation *Proxy* does not know which candidate is corresponding to this row.

After creating the ballot matrix, its columns – called *ballot columns* – are processed separately (as in the case of ThreeBallot). In the next step *Proxy* obtains a blind signature (BS) of *EA* under each ballot column. (A blind signature is necessary in order to prevent changing the ballot contents by *EA* at this moment). The voter enters ballot serial number (i.e. S_l), then *Proxy* unblinds the signature, and sends ballot columns with S_l to *EA*. Simultaneously, the voter requests one ballot column as a receipt. The receipt contains:

- a confirmation token $T \in \{A, B, C, D\}$ corresponding to the ballot column chosen,
- y - the ballot column itself,
- a value t such that $T = \text{sign}_{EA}(t, S_l)$; t is called a pre-token of T.

The ballot columns are now separated and published just like for Punchscan scheme, and then decrypted in a similar way. The number of votes for each candidate is counted like for ThreeBallot.

2 Security model

We need the following assumptions for integrity: :

1. we use cryptographically secure bit commitments, i. e. *EA* cannot find different values having the same commitment value,

2. the authority (*EA*) that creates the ballots is trusted to keep secret the serial numbers and permutation used on each ballot (we use the same ballot generation technique as used by the Punchscan system to achieve this property),
3. the list of registered voters is public, i. e. each voter can check if he participated in elections or not. This assumption is necessary to guarantee that *EA* cannot cast additional votes.

This scheme defends against remote, wholesale attacks on voters. The scheme is not resistant against physical, in-person attacks (physical coercion). Recall that the scheme from the previous chapter is resistant against such attacks.

There are assumptions for privacy:

1. the computer used by the voter will not record and transmit the voting session to *EA*,
2. the voter casts her vote unattended,
3. bit commitment scheme used cannot be broken.

3 Details of *Scratch, Click & Vote* scheme

3.1 The ballots and audit tables

The ballots are created by *EA*. In order to guarantee election integrity, *EA* generates audit tables P and R (see below for details). Each row of table P corresponds to a single ballot matrix (which is a set of columns with the same ballot serial number and the permutation of the candidates). Entries of R correspond to single ballot columns. R is based on the same idea as Punchboard used in Punchscan.

3.2 Table *P*.

The table P has 2 columns, called P1 and P2. It has $2n$ rows, where n is greater or equal to the maximum number of voters.

P1	P2
$\vdots$	$\vdots$
$S_l(i)$	$BC(i_A), BC(i_B), BC(i_C), BC(i_D)$
$\vdots$	$\vdots$

Example: Audit table P

The column P1 records the ballot serial numbers. The column P2 contains commitments to 4 pointers to the rows of table R. Say, if a serial number S is in P1, then in the same row, column P2 contains commitments:

$$BC(i_A(S)),\ BC(i_B(S)),\ BC(i_C(S)),\ BC(i_D(S))$$

to numbers $i_A(S)$, $i_B(S)$, $i_C(S)$, $i_D(S)$, where $i_X(S)$ is the row number such that row $i_X(S)$ of table R contains an entry for the column X of the ballot with the serial number S.

3.3 Table *R*.

The table R consists of three parts: the *starting part*, the *middle part* and the *final part*. Each part consists of a set of consecutive columns. R has $8n$ rows; this corresponds to $2n$ ballots and thus to $4 \cdot 2n$ ballot columns. There are two types of permutations used for constructing table R:

- ρ_1, ρ_2: permutations of rows of the R (i.e. permutations over $\{1, \dots 8n\}$),
- permutations π'_i, π''_i for $i = 1, \dots, 8n$ over $\{1, \dots, m\}$, where m is the number of candidates (i.e. 2 permutations per each of $8n$ ballot columns). These permutations have to be applied to ballot columns.

Each row i in the starting part of R is devoted to a single ballot column of some ballot (and for each ballot column from some ballot there is exactly one such a row of R). Let $W(i)$ denote the ballot column corresponding to the ith row of R. Then for each i, data concerning $W(i)$ are placed in:

- row i of the starting part,
- row $\rho_1(i)$ of the middle part,
- row $\rho_2(\rho_1(i))$ of the final part.

Moreover:

- the starting part of row i will contain the ballot column $W(i)$ as filled by the voter (the order of the candidates is determined by $\pi_i = \pi'_i \circ \pi''_i$, i.e. the entry for a candidate j is given in row $\pi_i(j)$,
- the middle part at row $\rho_1(i)$ will contain $W(i)$ permuted by $(\pi'_i)^{-1}$,
- the final part at row $\rho_2(\rho_1(i))$ will contain $W(i)$ permuted by $(\pi''_i)^{-1} \circ (\pi'_i)^{-1}$. Hence the marks of $W(i)$ will be permuted according to the standard ordering of candidates: $(\pi''_i)^{-1} \circ (\pi'_i)^{-1} \circ \pi_i = (\pi''_i)^{-1} \circ (\pi'_i)^{-1} \circ \pi'_i \circ \pi''_i = id$.

Below we describe the ith row of R. Let $i = \rho_1(j)$ and $i = \rho_2(\rho_1(k))$.

starting part (for $W(i)$)	middle part (for $W(j)$)	final part (for $W(k)$)
i $\widehat{\pi}_i$ $H(t(i))\widehat{t(i)}\,y(i)\widehat{\rho_1(i)}$	$\widehat{\pi'_j}$ $y(j) \ll (\pi'_j)^{-1}$ $\widehat{\pi''_j}$ $\widehat{\rho_2(i)}$	v

Organization of a row of the table R.

The starting part contains the following entries in row i (see the diagram above):

- i — the row index ($i \in [1, 8n]$),

- $\widehat{\pi}_i$ — a bit commitment to the permutation of candidates π_i used in the ballot containing $W(i)$,
- $H(t(i))$— a hash of a *confirmation pre-token* $t(i)$, which satisfies the condition
$$T(i) = \text{sign}_{EA}(t(i), S_l(i)) \ ,$$
where $T(i)$ is the confirmation token used in conjunction with $W(i)$, and $S_l(i)$ is the serial number of the ballot containing $W(i)$,
- $\widehat{t(i)} = BC(T(i), S_i)$ — a bit commitment to the ballot serial number $S_l(i)$ of the ballot containing $W(i)$, and to the confirmation token $T(i)$,
- $y(i) = [y_0(i), y_1(i), \ldots, y_{m-1}(i)]$ — a vector holding mark $\times$ on those positions l such that $W(i)$ contains the $\times$ mark in row l. Initially, during creation of table R, the vector $y(i)$ is empty. It becomes filled after casting a vote.
- $\widehat{\rho_1(i)}$ — a commitment to the value $\rho_1(i)$.

The middle part of R in row i contains the following entries:

- $\widehat{\pi'_j}$ — a commitment to the permutation of candidates π'_j, where $\pi_j = \pi'_j \circ \pi''_j$,
- $y(j) \ll (\pi'_j)^{-1}$ — the vector $y(j)$ permuted by $(\pi'_j)^{-1}$,
- $\widehat{\pi''_j}$ — a commitment to the permutation π''_j,
- $\widehat{\rho_2(i)}$ — a commitment to the permutation $\rho_2(i)$.

The final part of R in row i contains the vector v equal to $y(k)$ permuted by $(\pi'_k)^{-1}$ and then by $(\pi''_k)^{-1}$ (i.e., listed according to the standard ordering of the candidates).

3.4 Preparation of ballots and audit tables

The ballots and the audit tables P and R are created by *EA* in the following way:

1. ***EA* determines the election parameters:** the number of candidates m, the official list of candidates (with their official ordering), and an upper bound n on the total number of voters.
2. ***EA* chooses at random $2n$ serial numbers; for each serial number S:**
   - *EA* chooses at random a random permutation π,
   - *EA* chooses at random confirmation pre-tokens $t_A(S)$, $t_B(S)$, $t_C(S)$, $t_D(S)$ and computes confirmation tokens $T_A(S), T_B(S), T_C(S), T_D(S)$ according to the following equation:
$$T_X(S) := \text{sign}_{EA}(S, t_X(S)) \ \text{ for } X = A, B, C, D \ .$$

3. ***EA* creates audit table P:** For this purpose, *EA* chooses at random a permutation σ of $1, \ldots, 8n$. Then $\sigma(4j-3), \ldots, \sigma(4j)$ are assigned to the jth serial number $S_l(j)$. These numbers serve as pointers to the rows of the audit table R - and are called $i_A(S_l(j)), i_B(S_l(j)), i_C(S_l(j)), i_D(S_l(j))$. Then for each serial number $S_l(j)$, commitments to the values $i_A(S_l(j))$, $i_B(S_l(j))$, $i_C(S_l(j))$, $i_D(S_l(j)))$ are created and inserted in the row containing $S_l(j)$.
4. ***EA* prepares the audit table R:** For this purpose *EA* chooses at random permutations ρ_1 and ρ_2 of $1, \ldots, 8n$. For the jth serial number $S_l(j)$, its permutation π (on ballot columns) is assigned to the rows $i_A(S_l(j))$, $i_B(S_l(j))$, $i_C(S_l(j))$, $i_D(S_l(j))$ of the starting part of R. (i.e., $\pi_{i_A(S_l(j))}$, $\pi_{i_B(S_l(j))}$, $\pi_{i_C(S_l(j))}$, and $\pi_{i_D(S_l(j))}$) take the value π). Separately for each row i of R, *EA* chooses at random permutations π'_i and π''_i such that $\pi_i = \pi'_i \circ \pi''_i$.
5. Then the entries of R are filled according to the description from the previous subsection.

Finally, the ballots are printed so that their contents (the permutation of the list of candidates names, confirmation tokens and serial numbers) is hidden under a scratch layer.

3.5 The pre-election audit

As for Punchscan, the following steps are executed in order to check that the audit tables have been created honestly:

1. The Auditors pick at random a set AS of n ballots. The remaining ballots create a so called election set ES (and are not checked).
2. The contents of all ballots from AS is revealed, so in particular their serial numbers. Based on the serial numbers it is possible to indicate the rows of P corresponding to the ballots from AS.
3. *EA* opens all bit commitments from table P corresponding to the ballots from AS as well as all bit commitments from table R corresponding to the ballot columns of the ballots from AS.
4. The Auditors check whether the ballots and the entries in the audit tables were created correctly.
5. All ballots from the audit set AS are discarded; the ballots with serial numbers in ES are used for election.

In practice, the Auditors may confine themselves to controlling only a limited number of ballots from AS, and check more ballots on demand.

3.6 Coding Cards generation

The coding cards are prepared in an electronic form and are published (as commitments) on a webpage by the *Proxy*. Their correctness is checked in a standard way:

1. *Proxy* creates an audit table X in which it commits to coding card serial numbers S_r and positions of `Y`-marks on each coding card.
2. The Auditors select at random some number of coding cards to form an audit set (these coding cards are not used for elections).
3. *Proxy* opens all bit commitments from the cards of the audit set.
4. The Auditors check if the revealed coding cards have been created correctly.

3.7 Elections

The following steps are during vote casting:

Step 1: the voter obtains a ballot (e.g. by visiting certain authorities, from a special courier delivering the ballots at residence area, by certified mail services etc.). At the same time, identity of the voter is verified and the ballot is given to her own hands. Distribution of ballots is organized so that nobody knows who gets which ballot. Since the ballot information is covered with a scratch surface, this is easy.

Step 2: the voter fetches (still unused) coding cards from one or more Proxies (for convenience the coding cards can be printed).

Step 3: the voter peels-off the scratch-layer from the ballot.

Step 4: the voter logs in an election webpage run by a *Proxy* and authenticates herself.

Step 5: *Proxy* verifies voter's credentials.

Step 6: the voter chooses one of the coding cards and lays it next to the ballot.

Step 7: the voter clicks on the PC screen on radio buttons corresponding to her choice – that is, according to the permutation used for the ballot and alignment of `n` and `Y`'s marks on the coding card as described in Section 1.4.

Step 8: *Proxy* commits to voter's clicks, sends the commitment to *EA* and to the voter (so the voter can print it)[2].

Step 9: the voter enters S_r from the coding card used.

Step 10: *Proxy* transforms the voter's choice into ballot columns.

Step 11: *Proxy* obtains a blind signature from *EA* under each of the ballot columns (these signatures are then stored by *Proxy* for a post-election audit).

Step 12: the voter enters S_l.

[2] This commitment can be later used during investigation in the case if a fraud was detected.

Step 13: ***Proxy*** passes S_l and the ballot columns to *EA*.

Step 14: ***EA*** replies with a receipt of the ballot columns obtained and enters the obtained ballot columns into appropriate rows of the starting part of table R (but *EA* publishes them when the election are closed), *EA* publishes commitments to the ballot columns obtained from *Proxy*.

Step 15: the voter chooses a receipt (one of the four ballot columns). The receipt is obtained from *Proxy*.

3.8 Tallying

1. When the voting time is over, *EA* publishes voter's choices inserted into vectors $y(i)$ in the starting part of the table R. Then it computes the entries for the middle part of R:

   $$y(j) \ll (\pi'_j)^{-1} \ ,$$

   and for the final part:

   $$v := (y(k) \ll (\pi'_k)^{-1}) \ll (\pi''_k)^{-1} \ .$$

2. From the entries v in the final part *EA* calculates the tally: If the number of ballot columns is $4N$ (meaning that N votes have been cast) and there are together M marks $\times$ in row j of all ballot columns in the final part of R, then the number of votes cast for the jth candidate is $M - 2N$.

   All values computed above are published.

3.9 Post-election audit

Post-election audit is needed to verify if *Election Authority* performed vote casting and tallying steps according to the protocol.

Definition 1. *We say that a vote inserted into R table by Election Authority is* formally correct *if it is a correct vote for one of the candidates.*

Formally correct vote in l out of k race contains exactly l rows with three $\times$-marks and $k - l$ rows with two $\times$-marks. If a vote is formally correct it does not mean that it corresponds to a voter's choice – *Election Authority* might change her vote into a vote for a different candidate – during post-election audit such an attempt will be detected with high probability.

Definition 2. *We say that rows of the table R are* related *if they correspond to the same row of the P table.*

Post-election audit consists of three phases.

A *Election Authority* proves to *Proxy* that all ballot columns inserted into table R are correct, i.e.: blind signatures under ballot columns are correct[3]. Then *Proxy* checks if ballot columns are correctly inserted into R table:

1 *Election Authority* shows which ballot columns are related.

2 *Proxy* verifies:

a correctness of the commitments – links between P and R,

b that related ballot columns constitute formally correct vote.

B Global/public verification – integrity of table R and the election results is performed in public by the auditors (global/public verification). For this purpose the standard Randomized Partial Checking [12] procedure is executed for R (for the sake of simplicity of description we assume that n voters participated in the elections):

1 The auditors choose $2n$ rows[4] of R at random and request *EA* to open commitments $\widehat{\rho_1(i)}$ from these rows. Then for each row $\rho_1(i)$ in the middle part, for which $\rho_1(i)$ has been revealed, the commitment $\widehat{\pi'_i}$ is opened and it is checked that the ballot column from the starting part permuted by $(\pi'_i)^{-1}$ yields the ballot column in the middle part.

2 For each row j in the middle part, not pointed to by any revealed commitment $\rho_1(i)$, *EA* has to open the commitments to $\rho_2(j)$ and $\widehat{\pi''}$. Then the ballot columns in the middle part of row j permuted with π''^{-1} and the ballot column in the final part of row $\rho_2(j)$ should be equal.

C Local/personal verification – each voter can check if her ballot column corresponding to the receipt appears in the table R. This is possible, due to knowledge of the verification pre-token t, one can locate the right row containing $H(t)$. If it is missing or the contents of the ballot column disagrees with the receipt, then a fraud is detected.

3.10 Masked columns

Scratch, Click & Vote inherits many properties of Punchscan and ThreeBallot schemes. A receipt obtained by a voter is a column with k rows. In ThreeBallot scheme there are 2^k types of receipts. One may easily see that many ballot columns from the final ballot box are not compatible with a receipt – if a receipt and a column have more than one row with two $\times$ marks, then they do not come from the same voter. This phenomenon leads to Strauss'-like attacks [1,19,18,7] on receipts in ThreeBallot scheme. One of the main goals *Scratch, Click & Vote* was to eliminate this weakness.

[3] Blind signatures that were obtained in the Step 11 of the scheme

[4] R table has $8n$ rows, $4n$ rows were revealed during pre-election audit. $2n$ rows

As we have described in Section 1.2, already two countermeasures were implemented to avoid this problem:

- the probability distribution of possible ballot-columns was modified (now it is closer to the uniform one, dependance are weakened), by increasing the number of columns and the number of ×-marks,
- the list of candidates is permuted.

If permutations are used for reordering lists of candidates, then there are just $k + 1$ types of receipts (any two receipts with the same number of × marks are in a certain sense equivalent). This is sufficient to eliminate all Strauss'-like attacks when the attacker has access to a single receipt of a voter and all public data (with all other receipts).

The situation is much worse when the attacker obtains additional data from the *Proxy*. Note that *Proxy* knows not only receipts obtained by each voter but also each ballot column cast – this, in some cases may lead to efficient attack on privacy. The solution to this problem is to use so called *masked ballot columns*. The idea is that table R stores in its rows instead of *ballot columns* – k corresponding *masked ballot columns*. Simply, the jth *masked ballot column* of a given *ballot column* contains no × mark except for the row j, provided that the original ballot column contains a × mark in the row j. See an example of a ballot column and its masked versions:

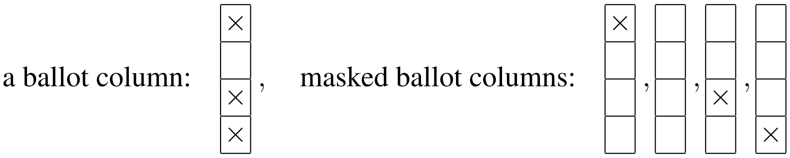


Fig. 6. Masked columns – instead of publishing k-row ballot columns, k masked ballot columns are published. The number of masked ballot columns of each kind is determined **uniquely** by the election result.

Let X be a ballot column and t be its pre-token. Then the jth masked ballot column for X in the starting part of R is marked by the value $H(t, j)$ (instead of $H(t)$, as it was for the first design). This enables the voter to check the entries of the bulletin board as before - however the voter has to look for k different hashes and k rows instead of one.

Checking integrity of R table is performed just as before, as well as vote counting: the number of × marks does not change, the number of votes is now the number of rows of table R divided by $4k$.

How do the masked ballot columns help to preserve anonymity? The key observation is that the number of masked ballot columns of each kind is determined **uniquely** by the election result. Therefore R table provides **no** additional information. So the Strauss' attack and any other attack based on the particular choice of ballot columns fail.

The same technique may be applied to ThreeBallot scheme.

3.11 Extensions

For the national elections it is strongly recommended to use scratch-off surface and provide secure and reliable channel for delivery of the cards. However, in some situations, one can use simpler and cheaper techniques. Moreover, for the elections in societies or companies one can assume that vote-buying would not be a problem. In this case, *SC&V* can be purely electronic: ballots and coding cards can be sent by email. Then, a voter prints them on one machine and casts a vote from a different one.

4 Usability issues - SCV with shifts

For certain voters, the scheme might be uneasy to use for at least two reasons. First, a voter needs to click next to every candidate. Second, a voter needs to find her candidate on a permuted list of candidates. So use of shifted lists of candidates (as for Prêt à Voter) can improve usability. Unfortunately, shifts have a negative impact on privacy.

A receipt obtained by a voter is an k-row column. If permutations are used for reordering lists of candidates, then there are $k+1$ types of receipts (any two receipts with the same number of $\times$ marks are in a certain sense equivalent).

The situation is different, if shifts are used. Then, the number of different types of receipts is equal to the number of k-bead necklaces with 2 colors which equals to (see [9]):

$$\mathcal{S}(k) = \frac{1}{k}\sum_{d|k} \varphi(d) 2^{\frac{k}{d}} \ ,$$

where $\varphi(k)$ is the Euler quotient function, i.e. for k-prime $\mathcal{S}(k) = 2^k/k$. In this case, in order to achieve privacy of votes, the number of voters has to be much higher than $\mathcal{S}(k)$.

If one wants to use shifts instead of permutation of candidates, one has to use masked-columns version of *Scratch, Click & Vote* instead of standard one.

k	2	3	4	5	6	7	8	9	10	11	12	13	14
$S(k)$	3	4	6	8	14	20	36	60	108	188	352	632	1 182

Fig. 7. SCV with shifts k – number of candidates; $S(k)$ – number of ρ_S different ballot columns when shifts are applied (equal to number of k-bead necklaces with 2 colors;

5 Security analysis

5.1 Integrity

Malware resistance Here we assume that the Alice's PC is dishonest, while *EA* and *Proxy* behave correctly. This corresponds to the case that Alice's PC is infected by malware.

Theorem 1 (Malware immunity). *The probability $P_{PC}(k_1, \ldots, k_l)$ that PC modifies voter's choice in l races with $k_1, \ldots, k_l$ candidates respectively without being caught on cheating bounded by*

$$\frac{1}{4}\frac{1}{3^{l-1}}\prod_{i=1}^{l}\frac{1}{k_i}$$

Proof. In order to manipulate voter's choice in the i-th race (change from Alice's choice to any other candidate, even a random one) the PC has to switch Alice's choice from `Y` into `n` in the row corresponding to the candidate chosen by Alice and at the same time, change `n` into `Y` in one of the remaining rows.
In order to do that, the PC has to guess which row corresponds to the chosen candidate; however success probability is only $p_1(k_i) = \frac{1}{k_i}$. Then, the PC has to choose one of the remaining rows and guess which one of the not chosen three columns corresponds to the mark `Y` – this succeeds with probability $p_2 = \frac{1}{3}$.
Probabilities $p_1(k_i)$ and p_2 are independent. We see that the probability of switching Alice's choice into another, correct choice in the ith race is $P_S(k_i) = p_1 p_2 = \frac{1}{3k_i}$. Probabilities of successful change of voter's choice in different races are independent, so the probability of a correct vote change in l races equals to: $P_S(k_1, \ldots, k_l) = \prod_{i=1}^{l} P_S(k_i)$.
However, even then, Alice can still detect a fraud, with probability p_3, by discovering that her receipt does not fit her choice. At least one of the ballot columns is modified during such a change, so $p_3 \geq \frac{1}{4}$ (and sometimes it is just one column), thus:

$$P_{PC}(k_1, \ldots, k_l) = (1 - p_3) P_S(k_1, \ldots, k_l) \leq \frac{1}{4}\frac{1}{3^{l-1}}\prod_{i=1}^{l}\frac{1}{k_i}.$$

Election Authority Misbehaviour in ballots' preparation and counting is limited by the pre- and post-election audits just like in the case of Punchscan (all unused ballots are opened).

Replacing ballot columns when inserting them to table R is risky, since the voter gets a receipt, which is one of her four ballot columns signed by *EA*. If the receipt disagrees with the contents of table R then one can catch *EA*. Recall that the ballot columns are signed blindly by *EA* before *EA* knows ballot's serial number (and thus the permutation). Moreover *EA* does not know which of the ballot columns is chosen for the receipt. Note that since the hash value of the pre-token is posted in R, the voter can prove which entry in the starting part of R corresponds to the ballot column from her receipt.

For the rest of this section we assume that the PC of Alice and the *Proxy* are honest.

Election Authority may try to influence on election results in three ways that lead to the situation when votes are decoded with different permutation π that was printed on the ballots:

1. to switch content of ballot columns in the starting part of table R so that at least some of related ballot columns are not formally correct votes – this situation can be detected in the phase A of the post-election audit;
2. to switch content of ballot columns in the starting part of table R so that all related ballot columns are formally correct votes (wrong ballot column insertion during vote casting) – this situation can be detected by personal verification (phase C) of the post-election audit);
3. not to transform ballot columns according to ρ_1 or ρ_2 (or π' or π'') (wrong vote counting during tallying) – this situation can be detected in the phase B of the post-election audit.

The following two theorems give us insight about the integrity of the elections. Both theorems assume that underlying commitment scheme is not breakable for *Election Authority* before post-election audit is done.

Lemma 1. *If EA modifies $\times$-marks of the related ballot columns so that they are formally correct vote then this situation can be detected only by personal verification.*

If EA modifies related ballot columns in R so that they are not formally correct vote then this situation can be detected in the phase C of the post-election audit.

Proof. *Election Authority* can change election outcome up to $2k + 1$ by modifying $\times$-marks in ballot columns of (only) one ballot. But then of course, *EA* must to adjust $2k+1$ ×-marks in other ballot columns because number of votes must agree with the number of votes cast.

The post-election audit require to reveal data corresponding to half of the rows of the P table – links between P and ballot columns of the same ballot. If *Election Authority* modifies k of the related ballot columns so that they are not formally correct then it is detected post-election audit with probability $1 - \frac{1}{2^k}$.

If *Election Authority* modifies related ballot columns so that they are formally correct then post-election audit does not detect this[5]. In this case, *Election Authority* is caught on cheating if and only if a voter finds out that her receipt does not corresponds to data published in R.

Theorem 2. *Assume that Election Authority have manipulated m ballot columns in the starting part of R. If v voters perform personal verification then P_S – the probability that Election Authority will not be caught on cheating is bounded by*

$$\prod_{i=0}^{v-1}\left(1 - \frac{m-i}{4N}\right)$$

where N is the number of votes cast.

Proof. For N votes cast, there are at most $4N$ ballot columns that contain at least one x-mark.

According to the Lemma 1, *Election Authority* by modifying single ballot (=four ballot columns) can influence only on one vote (without increasing exponentially probability of being caught on cheating).

Probability that any of m manipulated ballot columns is <u>not</u> verified by at least one of the v voters is equal to:

$$P_{v,m,N} = \prod_{i=0}^{v-1} \frac{4N-m-i}{4N} \leq (1 - \frac{m}{4N})^v$$

So the probability that *Election Authority* will not be caught on cheating is:

$$P_S = 1 - P_{v,m,N} = 1 - \prod_{i=0}^{v-1} \frac{4N-m-i}{4N} \geq 1 - (1 - \frac{m}{4N})^v$$

For N large enough, we obtain $P_S \geq 1 - e^{-\frac{vm}{4N}}$.

For instance, if *Election Authority* wants to subvert elections by modifying 1% of the 100 000 votes cast, it succeeds with probability 0.006 737 2 000

[5] because related ballots are formally correct

Theorem 3. *The probability $P_{M,F}(m)$ that Election Authority modifies m ballot columns in the middle or final part of the R table without being caught on cheating during post-election audit is bounded by $P_{M,F}(m) \leq \frac{1}{2^m}$.*

Proof. Assume that *Election Authority* does not follow the protocol and manipulates m_1 ballot columns in the middle part. That is, *EA* does not transform m_1 ballot columns from the starting part of R-table to the middle part according to ρ_1 and $\{\pi'_i\}_{i=1}^n$. Then, it succeeds with probability $P_M(m_1) = \frac{1}{2^{m_1}}$ (this situation will be revealed while half of the links between starting and middle parts are shown during post-election audit).

If *Election Authority* does not follow the protocol and manipulates $m - m_1$ ballot columns in the final part – it does not transform from the middle part of R-table to the final part according to ρ_2 and $\{\pi''_i\}_{i=1}^n$. Then with probability $P_F(m - m_1) = \frac{1}{2^{m-m_1}}$ such a situation will be revealed during post election audit.

Let us notice that any modification of transformations between starting and middle parts and between middle and final parts that concern ballot columns corresponding to the same row of R are revealed with probability 1 since one of the two transformations for each such a rows are revealed. So one can assume that modifications of transformations between different parts are disjoint and thus independent. So $P_{M,F}(m) \leq P_M(m_1) \cdot P_F(m - m_1)$

Proxy Now we assume that *Proxy* is dishonest, while *EA* and the PC of Alice behave correctly.

Proxy commits to the voter's clicks before it knows S_r, so *Proxy* cannot change voter's choice.

If *Proxy* changes S_l in order to change the permutation of a ballot, then Alice obtains a confirmation token that is different from the one stated on her ballot. Thus it will be detected immediately by Alice.

5.2 Privacy

Malware resistance Even if Alice's PC sends all information it is aware of to an attacker, he would be unable to determine the choice of Alice. Indeed, the attacker neither knows the configuration of Y's on the coding card nor the permutation used on the ballot.

EA The situation is like for Punchscan: If *EA* knows which ballot was used by Alice, then it knows the vote cast by Alice. So it is crucial to apply appropriate procedures of ballot distribution. Keeping the sensitive information under scratch surface is a good solution - the ballots can be mixed before distribution, becoming thus indistinguishable. Also, it is crucial that voters never sends ballot information directly to *EA* - all communication must go through *Proxy*.

Proxy The assignments of `Y`'s and `n`'s are known to *Proxy*, so *Proxy* knows the row corresponding to the voter's choice. However, *Proxy* does not know the permutation used in the ballot, so it cannot link the vote with any particular candidate. *Proxy* may attempt to perform Strauss'-like attack but if masked-ballots (Section 3.10) are used then it is infeasible.

External observer's point of view Here, we assume that the observer Charlie is not physically present during casting a vote and does not control the PC used by Alice. We assume that Alice casts the vote and then passes to Charlie (e.g. by mail or fax):

- the ballot,
- the coding card,
- the receipt,
- information on which fields have been clicked.

Of course, Charlie has access to the bulletin board.

The key point is that Alice could use a coding card different from the one she shows to the observer – the receipt does not contain S_r. So, the situation of Charlie is much different from the situation of a *Proxy*: Charlie obtains only one ballot column (receipt) and cannot be sure if the coding card obtained was really used.

5.3 Vote selling

In case of *SC&V* a voter is authenticated electronically by the *Proxy*. The authentication protocol should guarantee that the voter would not risk transmitting her electronic identity to the buyer. (In this way *SC&V* becomes superior over postal procedures, for which transferring a ballot to a buyer cannot be prevented.) This holds for instance, if the voter is using an electronic ID card or ID codes that are used also for other purposes (like submitting a tax declaration).

Even if Alice casts the vote herself, she can record the whole voting session and present it to the buyer together with the ballot and the coding card used. The ballots have a non-electronic, paper form, so they can be presented to the remote buyer as electronic copies. However, the scan of the ballot can be manipulated and the coding card presented needs not to be the one actually used.

Things become more complicated for the buyer, if the authentication protocol is based on a zero-knowledge protocol – then the buyer cannot even be sure that the voter is casting a vote unless he is controlling directly the voter's PC.

The only thing that the buyer can be convinced about is the receipt and the matching entries in the bulletin board. However, at this moment we fall back to the case of the external observer considered above.

5.4 Decoy service

A decoy service can be optionally launched in order to make vote-buying and coercion harder. A voter can visit a webpage of a decoy service and download fake coding card with any given serial number and arbitrarily chosen arrangement of `Y`'s and `n`'s. Then, the voter may present such a coding card to a coercer or a vote buyer together with a receipt and ballot to prove that she voted in a certain way.

The idea of a decoy service was mentioned by many designers of e-voting schemes, sometimes replacing decent security mechanism. Nevertheless, it can be view as an important second line defence against vote buying and coercion and should be recommended for e-voting implementations.

6 Final remarks

SC&V allows for secure and verifiable vote casting over the Internet with unconditional integrity. Privacy is preserved with the assumption that *Proxy* and *Election Authority* do not collude.

A voter cannot prove how she voted unless vote-casting is physically supervised by an adversary (it is not the case in remote versions of Punchscan [16], where voter may remotely sell her vote).

Online vote-selling is almost impossible. In order to buy a vote, a buyer needs to obtain:

- the record of a voting session from the voter's computer (the serial numbers of ballot and coding card, and the voter's choices),
- ballot and coding card used.

Even if the voter's PC is infected by viruses, her choice remains secret. Moreover, any attempt of modification of voter's choice is detected with high probability.

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